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Aftershock series of event February 18, 1996: An interpretation in terms of self-organized criticality

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Abstract. An aftershock interevent time series, initiated on February 18, 1996, in the eastern Pyrenees was analyzed. The threshold detection magnitude was set at 1.9, and the series was assumed to be complete for an interval of 77 days. The original time series does not fit Omori's law, probably because of sudden changes in the rate of occurrence, interpreted as an increase in the production rate. When the recorded interevent time series is classified in terms of leading aftershocks (those that satisfy a relaxation process) and cascades (those occurred at a nearly constant rate), the new time series of the leading aftershocks fits Omori's law quite well, with $p = 0.94$. Interpreted in terms of Dietrich's model, the series of leading aftershocks correctly predicts a return time for the main shock of the order of 50 years. To interpret the series of cascades, a minimalist, self-organized critical model was used. Although it is very simple, the model correctly reproduces the two-level structure in the observed time series, that is, the sequence of leading aftershocks and a cascade sequence emerging from each aftershock. This model may be given physical justification in terms of the *Cochard and Madariaga* [1996] nucleation model.

1. Introduction

Large earthquakes are in general followed by a series of events of lower magnitude, localized at the same place and with similar focal mechanisms. These events are called aftershocks. In their recent book on global seismology, *Lay and Wallace* [1995, p. 385] refer to aftershocks as follows:

> Nearly all large earthquakes are followed by a sequence of smaller earthquakes, known as aftershocks, which are apparently related to the fault plane that slipped during the main event. The large earthquake, known as the main shock, introduces a major stress adjustment to a complex system by its sudden slip. Regions between the rupture zone, or adjacent to it, may require adjustment to the new stress state in the source volume, thus generating aftershocks. Aftershocks typically begin immediately after a main shock and are distributed throughout the source volume. Typically, the frequency of occurrence of aftershocks decays rapidly following Omori's law,

according to which the rate decay of aftershock sequences is proportional to t^{-1}, where t is the lapse time from the main shock. With respect to the origin of aftershocks, *Lay and Wallace* [1995, p. 385] state "Aftershocks are clearly a process of relaxing stress concentration introduced by the rupture of the main shock." In other words, we can contemplate the series of aftershocks as a nonstationary (relaxation) point process that presents some kind of clustering. Since the end of the last century it has been known that the decay of aftershock activity is well represented by Omori's law, one of the few firmly established empirical laws in seismology. For a historical review on Omori's law and its application, see *Utsu et al.* [1995, and references therein]. As noted by the above mentioned authors, Omori's law is unique in the sense that it displays a power law dependence on time with no presence of any characteristic timescale as a relaxation time.

However, the fit of observed aftershock series to Omori's law is only approximate, since the observed series does not show a smooth relaxation, that is, a gradual decrease of the event rate. A sudden increase in the event rate can be explained in terms of the occurrence of a new series of aftershocks, beginning with an event of larger magnitude than the preceding events and thus initiating a branching process. Then, the observed series can be simulated as a superposition of several Omori's series shifted in time. This superposition of aftershock series, known as epidemic type aftershock sequence (ETAS), was studied in detail by *Ogata* [1988].

On the other hand, observed aftershock series have also been fitted by several authors to other relaxation

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laws, exponential laws for instance [e.g., *Kisslinger*, 1993; *Marcellini*, 1995], but no physical models have been developed to justify the alternative relaxation laws, as in the case of Omori's law.

The present paper has been written under the assumption of the validity of Omori's law, considered a universal feature of the aftershocks occurrence. Once accepting the validity of Omori's law, the results presented in this paper should be understood as the fine structure of the process of nucleation.

The purpose of the present paper is to look for an explanation of the interevent time series of the aftershocks that followed the event of February 18, 1996, in the eastern Pyrenees. Apparently, the series of aftershocks under study do not follow Omori's law, due to the presence of sudden changes in the occurrence rate and to a lack of large events that would justify the use of an ETAS model. In order to explain these discrepancies, a different point of view has been adopted. The observed series has been separated into two classes of events, the first including those that strictly follow a relaxation process, the leading aftershocks, and the second containing the rest. The latter are termed cascades and can be described as series of events taking place at a higher rate. Because of the difficulties of applying two different friction laws in order to explain the observed complex time behavior, we have tried to give an alternative explanation in terms of self-organized criticality (SOC), which, as will be shown, correctly predicts the observed time series of occurrence of aftershocks.

The plan of the paper is as follows. In section 2 we briefly summarize the physical basis underlying the universality of Omori's law. Section 3 is devoted to the analysis of the observed series of aftershocks and its usual interpretation in terms of directly fitting Omori's law to the raw data. In section 4 a new approach to the study of aftershocks is suggested, based on the separation between leading aftershocks and cascades. In section 5 we introduce a minimalist model based on SOC behavior, explain our new approach, and summarize the main numerical results. Finally, section 6 is devoted to the discussion of the methods we have applied and the results.

2. Physical Basis of Omori's Law

The modified Omori's law [*Utsu*, 1961], cited by *Utsu et al.* [1995], is expressed as

$$n(t) = \frac{K}{(t+c)^p}, \tag{1}$$

where $n(t)$ is the occurrence rate of aftershocks, t is time, and K, c and p are constants. The cumulative number of aftershocks $N(t)$, defined as $N(t) = \int_0^t n(s)ds$ is

$$N(t) = \frac{K\left[c^{(1-p)} - (c+t)^{(1-p)}\right]}{(p-1)}. \tag{2}$$

The physical basis for the power law decay of aftershocks with time have been established through two different points of view: the continuous models developed by *Yamashita and Knopoff* [1987], *Shaw* [1993], and *Dietrich* [1994] and the discrete models developed by *Burridge and Knopoff* [1967] and *Ito and Matsuzaki* [1990], among others (for a review see *Shaw* [1993]).

Yamashita and Knopoff [1987] assumed, first, that the stress corrosion cracking is the physical mechanism for the delayed fracture in aftershocks and took into account the geometrical complexity of earthquake fracture zones, that is, aftershocks cannot occur without the presence of stress inhomogeneities, which cause the highly irregular slip during rupture. As a second assumption, they introduce beforehand a power law distribution of crack sizes with a power law for the rate of growth of the cracks. Under both hypotheses, the probability density of occurrence time of aftershocks is found to obey Omori's law.

Shaw [1993] and *Dietrich* [1994] describe the occurrence of an event by means of deterministic dynamics for the nucleation. The term earthquake nucleation is used to describe the process that leads to the initiation of an earthquake instability at some specific place and time. *Shaw* [1993] attributes the distribution of time delays of aftershocks to the acceleration of stress during nucleation and to the fast redistribution of stresses during an event and assumes a nucleation velocity proportional to a power law. On the other hand, *Dietrich* [1994], motivated by laboratory friction experiments, formulates the nucleation of an event in terms of a general nonlinear friction law and assumes that the earthquake rate is due to the elastic stress change associated to prior earthquakes.

Dietrich [1994], in deriving his model for the rate of earthquake occurrence, presents a specific prediction of the model: the mean earthquake recurrence time. This parameter can forecast the average time between large events in the zone under study. The seismicity rate as a function of time after the stress step is expressed as

$$R = \frac{r\dot{\tau}/\dot{\tau}_r}{\left[\frac{\dot{\tau}}{\dot{\tau}_r}\exp\left(-\frac{\Delta\tau}{A\sigma}\right) - 1\right]\exp\left[-\frac{t}{t_a}\right] + 1}. \tag{3}$$

where R is the seismicity rate, r the reference seismicity rate, $\dot{\tau}_r$ and $\dot{\tau}$ the stressing rate prior to and following the stress step, $\Delta\tau$ the earthquake stress change, A a fault constitutive parameter, σ the normal stress, t time and t_a the characteristic relaxation time for seismicity to return to the steady state, that is, the aftershock duration. Equation (3) gives Omori's law for $t/t_a < 1$. *Dietrich* [1994] shows that the mean earthquake recurrence time t_r can be approximated as

$$t_r = t_a \frac{-\Delta\tau}{A\sigma}. \tag{4}$$

If we define

$$T = \frac{\dot{\tau}}{\dot{\tau}_r}, \qquad B = \left[\frac{\dot{\tau}}{\dot{\tau}_r}\exp\left(-\frac{\Delta\tau}{A\sigma}\right) - 1\right], \qquad C = t_a$$

then (3) can be integrated to obtain the cumulative function

$$F(t) = TC \ln\left[\frac{\exp(t/C) + B}{1 + B}\right], -1 \leq B \leq 0 \qquad (5)$$

which can be fitted to data to obtain t_a and t_r.

A different approach for the study of earthquake occurrence was devised by *Burridge and Knopoff* [1967], who used a one-dimensional (1-D) block-spring model to simulate stick-slip rupture and showed that aftershocks occur if linear viscous friction is introduced; even in this 1-D case, the decay of aftershock activity is approximated better by Omori's formula than by a simple exponential decay. The decay law of aftershock activity, as predicted by the Burridge and Knopoff model, critically relies on the friction law. This model was further extended to 2-D by several authors. *Nakanishi* [1992] was able to obtain in a natural way the series of aftershocks following a main shock and obeying Omori's law, whereas the series of main events obey the Gutenberg-Richter law; this model consists of a two-dimensional Earth's crust which is assumed to be driven by a viscous fluid flow under the crust. In all these models, Omori's power law is obtained under the hypothesis of power law distribution of fields or in terms of a nonlinear friction law, both assumptions supported by observational evidence.

From another point of view, *Bak and Tang* [1989] modeled the seismicity as a critical phenomenon and demonstrated that slowly driven dynamical systems with many degrees of freedom (such as the block-spring models) may naturally self-organize close to a critical state of the system [*Bak et al.*, 1988]. In the present case, we would consider the crust as a dynamical system that slowly accumulates stress. This stress will later be dissipated in the form of avalanches without a characteristic size. Each sudden avalanche is assimilated to an earthquake, and the lack of a characteristic scale accounts for the Guttenberg-Richter law.

The behavior of such systems, known as self-organized critical models, is usually simulated by means of a cellular automaton. The simplest physical model for self-organized criticality is the paradigmatic pile of sand: grains of sand are randomly dropped on the top of the pile until the slope attains the critical angle of repose. At this point, the critical state has been reached and any additional sand grain will trigger sand slides (avalanches) of various sizes. The frequency-size distribution of sand slides has been found to obey the Guttenberg-Richter law. In a recent paper, *Bak et al.* [1994] showed that SOC models and block-spring models can be directly related. As in the case of the Burridge and Knopoff model, early SOC models were not able to spontaneously generate aftershocks unless some modifications are introduced into the model, as, for example, that of *Ito and Matsuzaki* [1990]; these authors assimilate the occurrence of aftershocks to what they called a model of entropy relaxation, according to which the main shock will disturb the strain distribution, instead of the stress distribution, as considered in the other models. *Barriere and Turcotte* [1994] considered a 2-D cellular-automaton model with a fractal distribution of sizes for the grid of boxes; their model triggered aftershocks that did not obey, however, obey Omori's law. Y. Huang et al. (Precursors, aftershocks, criticallity and self-organized criticality, http://xxx.lanl.gov/abs/cond-mat/9612065, 1996), modified Barriere and Turcotte's model by adding the characteristics of the sand pile model of *Bak and Tang* [1989]; in this model the big earthquakes are followed by aftershocks that do obey Omori's law.

3. Data

On February 18, 1996, a local magnitude $M_L = 5.2$ earthquake occurred in the eastern Pyrenees. According to *Rigo et al.* [1997], the focal parameters are as follows: origin time = 0145:45 UT, epicentral location =3D N42°47.81′ - E2°32.30′, with a focal depth of 8 km. In the following two months, more than 500 aftershocks were recorded by the French permanent Pyrenean seismological network. An exhaustive report of the main shock and the largest aftershocks is given by *Rigo et al.* [1997].

The series of aftershocks that followed this event was recorded at the three-component continuous broadband seismic station of the Tunel del Cadi [*Vila*, 1997], located at about 80 km SW of the epicentral area. Figure 1 shows the location of the main shock and the Cadi seismic station (CAD). After a careful visual inspection of 3 months of records (more than 20 Gb of data), a series of aftershocks (that we strongly believe to be complete for a threshold magnitude of 1.9) was retrieved. The series consisted of 337 events, spanning a lapse time of 1846 hours (77 days, from February 18 to May 5, 1996) and with magnitudes ranging from 1.9 to 3.8. To assign magnitude to the events for which agency information is not available, we derived a particular magnitude law, obtained through a nonlinear fit of the amplitudes of our records to the M_L values given by the Laboratoire de Detection et Geophysique French Agency.

The cumulative series of aftershocks (solid circles) is shown in Figure 2, along with the amplitudes of the events, arbitrarily normalized to 300, to be compatible with the scale of the cumulative number of events. The sudden change in slope of the cumulative curve at 300 hours is striking (see Figure 2a); this change is not due to incompleteness of the series, and from the point of view of the amplitude of the events, there is no specific characteristic, nor any relevant event, that justifies this sudden change in the event rate. Because of this different behavior, from now on we will restrict our attention to the series defined by the first 300 hours (corresponding to 13 days, from February 18 to March 2) with a total of 308 events, as displayed in Figure 2b. A surprising feature of this series is the change in concavity of the cumulative curve, not correlated with any significant event, suggesting an increase in the rate of occurrence not justified by any relaxation process. Fig-

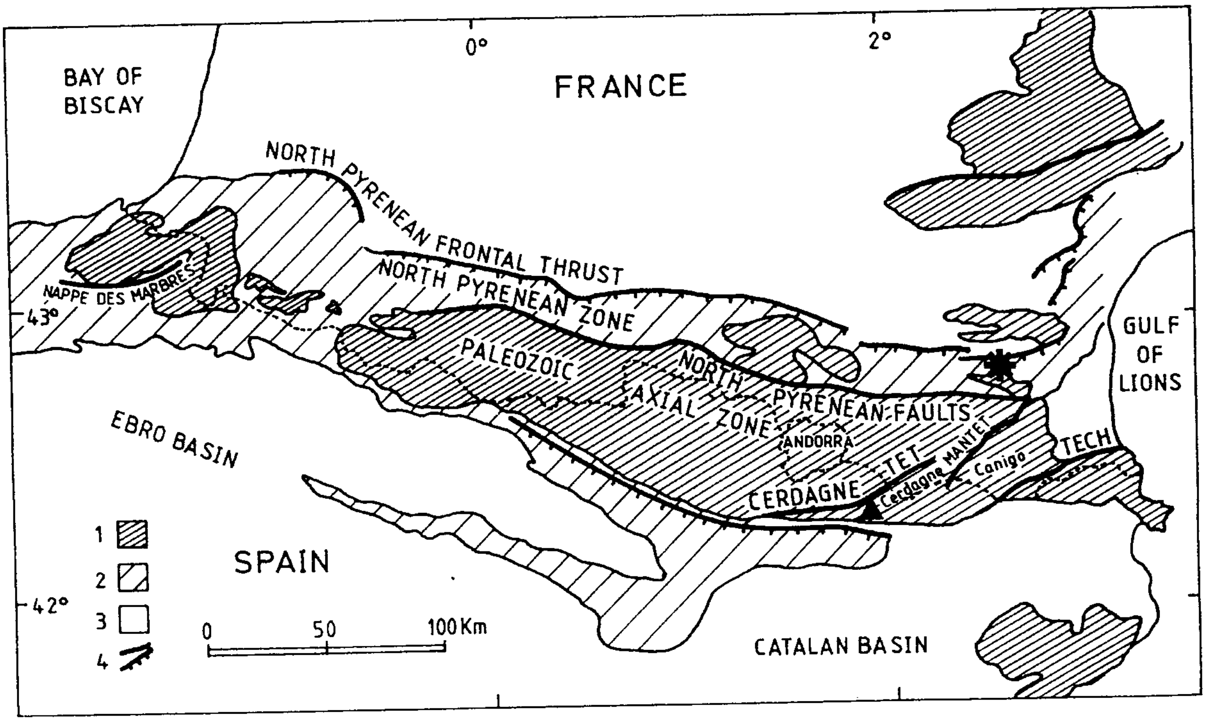

Figure 1. Overall view of the Pyrenees: 1, Paleozoic outcrops; 2, Mesozois and Eocene materials; 3, Neogene sediments; 4, Faults and thrusts. The location of the seismic station is shown by a triangle, and the large asterisk indicates the epicentral area.

ure 2c displays a detailed view of two series of events with negative concavity, once defined the positive concavity as that corresponding to a decreasing rate of occurrence, as predicted by Omori's law. Data from Figure 3b have been fitted to the cumulative number of aftershocks equation (2). The best fit has been obtained by splitting the aftershock series into two, the first for a time interval of 0 – 100 hours and the second for 140 – 300 hours; no fit can be obtained for the interval 100 - 140 hours. Results are displayed in Figure 3a and Table 1. The values of p are abnormally low [*Utsu et al.*, 1995] and, as already stated, there is no apparent reason for the change of activity from 100 to 140 hours.

With the aim of obtaining a better fit, an attempt has been made to fit Omori's law to a new series of aftershocks, constructed with a higher magnitude threshold. Figure 3b shows the fit of the cumulative number of aftershocks for a magnitude threshold of 2.6; the total number of events has now been reduced to 33, and the p value has been increased to 0.75, still too low. The fit is good at the beginning and at the end of the series, but between 20 and 100 hours we can still observe an increase in the occurrence rate.

Following *Dietrich* [1994], the recurrence time t_r has been computed through the fit of (5) to the two time intervals presented in Table 1, obtaining $t_r = 6.5$ years for the first interval and $t_r = 7.9$ years for the second interval. A comparison with a seismic catalog of the zone under study [*Suriñach and Roca*, 1982] reveals that both recurrence times are 1 order of magnitude too low with respect to that deduced from the seismic catalog, of the order of 50 years.

4. New Approach to the Study of Aftershocks

In the previous section we have seen a lack of fit of our recorded aftershock series to Omori's law and that the changes in concavity of the curve defined by the accumulated number of events cannot be correlated with the presence of any large aftershock able to generate a secondary series of aftershocks. On the other hand, Omori's law has a physical justification in terms of a relaxation process, implying that the time interval between successive events is a monotonically increasing function.

The interpretation of Omori's law as a relaxation process suggests a way to separate the observed series of aftershocks into two classes: class A, for those events that follow a relaxation law and class B for those events that do not. The criterion to assign the events to classes A or B is the following: if the interval of time Δt_i between events i and $i-1$ is strictly larger than the interval of time Δt_{i-1} between events $i-1$ and $i-2$, then event i belongs to class A, otherwise it belongs to class B.

Events belonging to class A are termed leading aftershocks, whereas those belonging to class B are termed cascades. Figure 4a shows the series of aftershocks classified as leading events (solid circles) and cascades (points); note that a cascade is initiated by a leading aftershock. Figure 4b displays the fit of the series of leading aftershocks to Omori's law: the fit is now very good, and the value obtained for the exponent is $p = 0.94$. Figure 5a shows the series of cascades, in which the first term of each cascade is a leading aftershock. Two char-

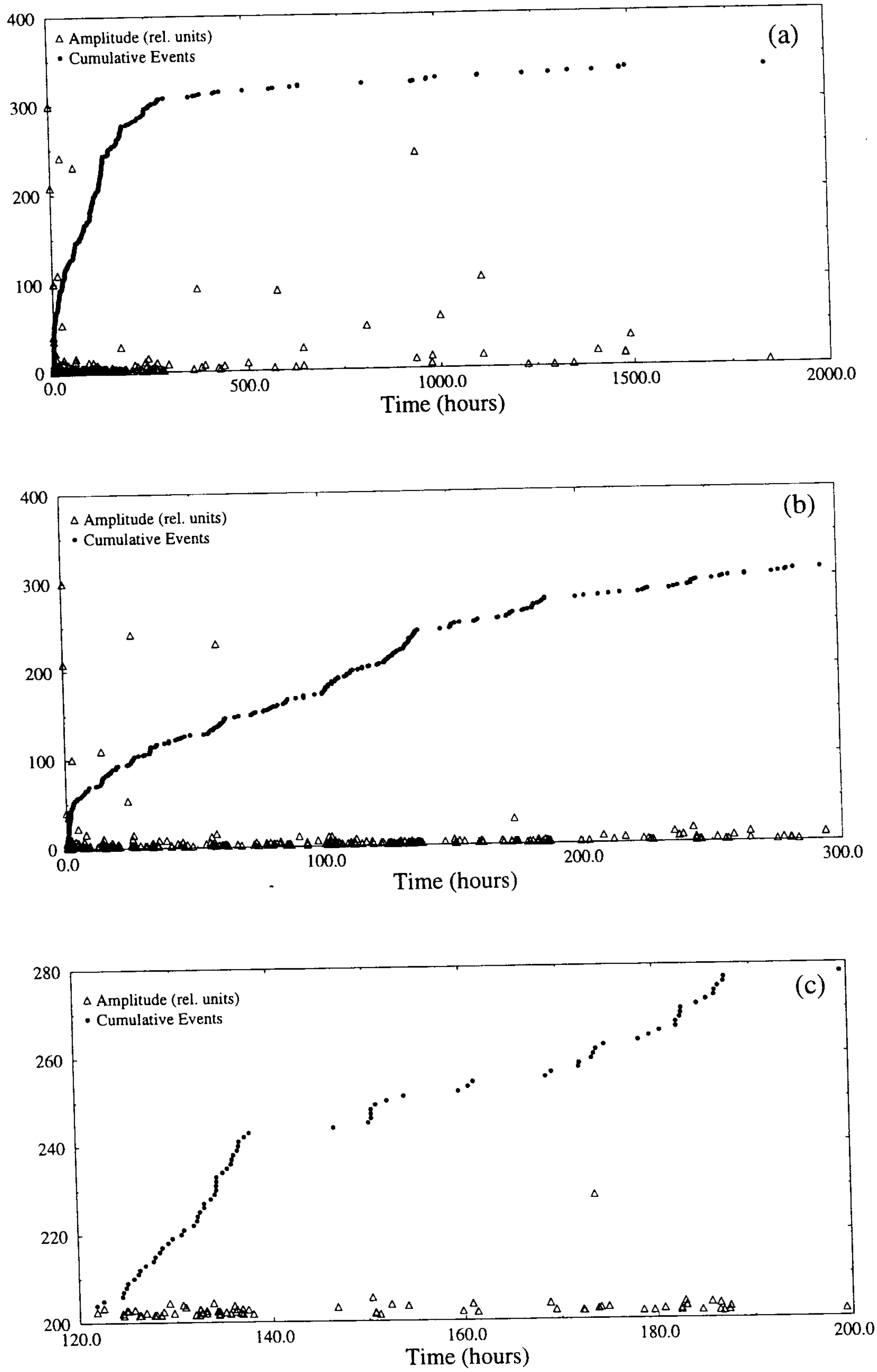

Figure 2. Cumulative series of aftershocks (solid circles) and amplitudes (triangles, in arbitrary relative units). (a) displays the Cumulative series of events for an interval of 1900 hours. Note the abrupt decrease of the rate of occurrence at about 300 hours, not correlated with any specific change in the amplitudes. (b) First 300 hours of activity along with the relative amplitudes. Sudden changes of activity appear, not correlated with any change of the amplitudes. (c) Some details of the change of concavity of the cumulative curve.

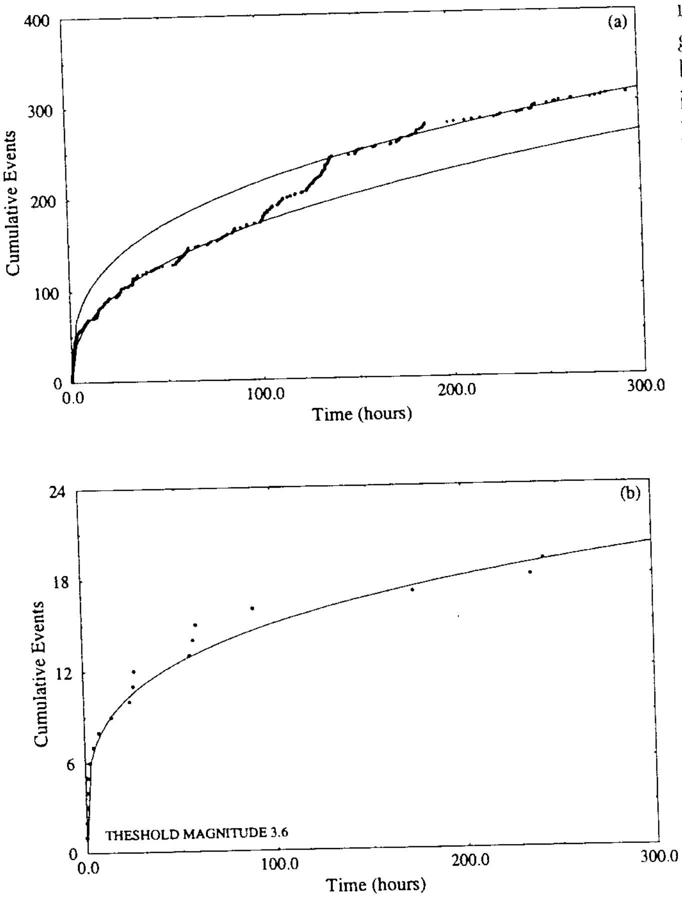

Figure 3. Fit of the first 300 hours of activity to Omori's law. (a) We can see that no single curve can correctly fit the observations. (b) Detection threshold has been raised to a magnitude of 2.6. The fit has improved, but there is still considerable scatter.

acteristics are shown in this figure: (1) the cascades are in general well approximated by straight lines and (2) their corresponding slopes decrease with time. Since the first term of each cascade is a leading aftershock, we can observe an increment in the time elapsed between successive cascades, in good agreement with a relaxation process. Figure 5b displays the slope of the cascades versus the occurrence time of the leading aftershocks; the slopes fit a power law, defined as $y = 23.4 \times x^{-0.71}$. Up to now, no interpretation has been found for this power law behavior.

In terms of the *Dietrich* [1994] model, aftershocks are caused by the steplike change of stress that occurs at the time of the main shock. When the mean earthquake recurrence time t_r has been computed as predicted by Dietrich's model for the series of leading aftershocks, the value obtained is $t_r = 47$ years, in good agreement with the observed seismicity of this zone [*Suriñach and Roca*, 1982; C. Olivera, personal communication, 1997].

The occurrence of the series of aftershocks could be qualitatively explained in terms of an asperity model: a leading aftershock would initiate the breaking of an asperity that would proceed discontinuously, at steps, each one originating an event or a cascade. If we take into account only the series of leading aftershocks, a good fit to Omori's law is obtained. Hence they can be interpreted in terms of Dietrich's model, thus obeying a state-dependent friction law. However, if Omori's law is not able to explain the occurrence of cascades, a different friction law should be derived to explain their occurrence, that is, the appearance of events at a nearly constant time, in other words, the occurrence of periodic events which imply a constant rate friction law. This could indeed be the case inside an asperity of finite dimensions, implying two different friction laws, one responsible for the initiation of the rupture of an asperity and the other describing its rupture. The time elapsed for the breaking of an asperity, which depends on how heterogeneous this asperity is, is relatively short. For example, for the longest cascade, initiated at 125.7 hours after the main shock and consisting of 33 events (see Figure 5a), the rupture time is 1.7 hours. The decrease in the slope of the cascades, a measure of the slip velocity of the asperity, could be explained by taking into account that the slip velocity is a function of the stress drop, and the average stress accumulated in the source volume decreases in time, in the form of radiated seismic waves or lost as irreversible processes. The lack of previous observations of cascades is probably due to the magnitude threshold currently used, normally higher than 3, compared to the actual threshold of 1.9. It is worth to point out that the magnitude of the events that define the cascade is less than 2.5.

From the point of view of a continuous model, it is a huge task to quantitatively model the generation of a relaxation process able to reproduce the characteristics displayed by the observations. Hence a different point of view has been adopted, that of systems at the critical point, and a simple explanation of the geometry of the time series, considered as a point process, can be given in terms of SOC.

5. Minimalist Model

In this section we will provide an explanation of the geometry of the interevent time series of aftershock occurrence (leading aftershocks and cascades) under the hypothesis that this time series can be considered as a nonstationary point process. The geometrical characteristics of the observed time series, as displayed in Figure 2b, consist of successive changes in concavity. As already stated, the series of events defining a region of negative concavity has been termed cascade, and the first event of each cascade is the leading aftershock. The series of leading aftershocks follows a relaxation process that obeys Omori's law, whereas the series of

Table 1. Fit of the Aftershock Series to Omori's Law

Time Interval	K	c	p
0 – 100	8.6 ± 0.2	0.2×10^{-4}	0.56 ± 0.01
140 – 300	13.2 ± 0.2	0.2×10^{-4}	0.64 ± 0.01

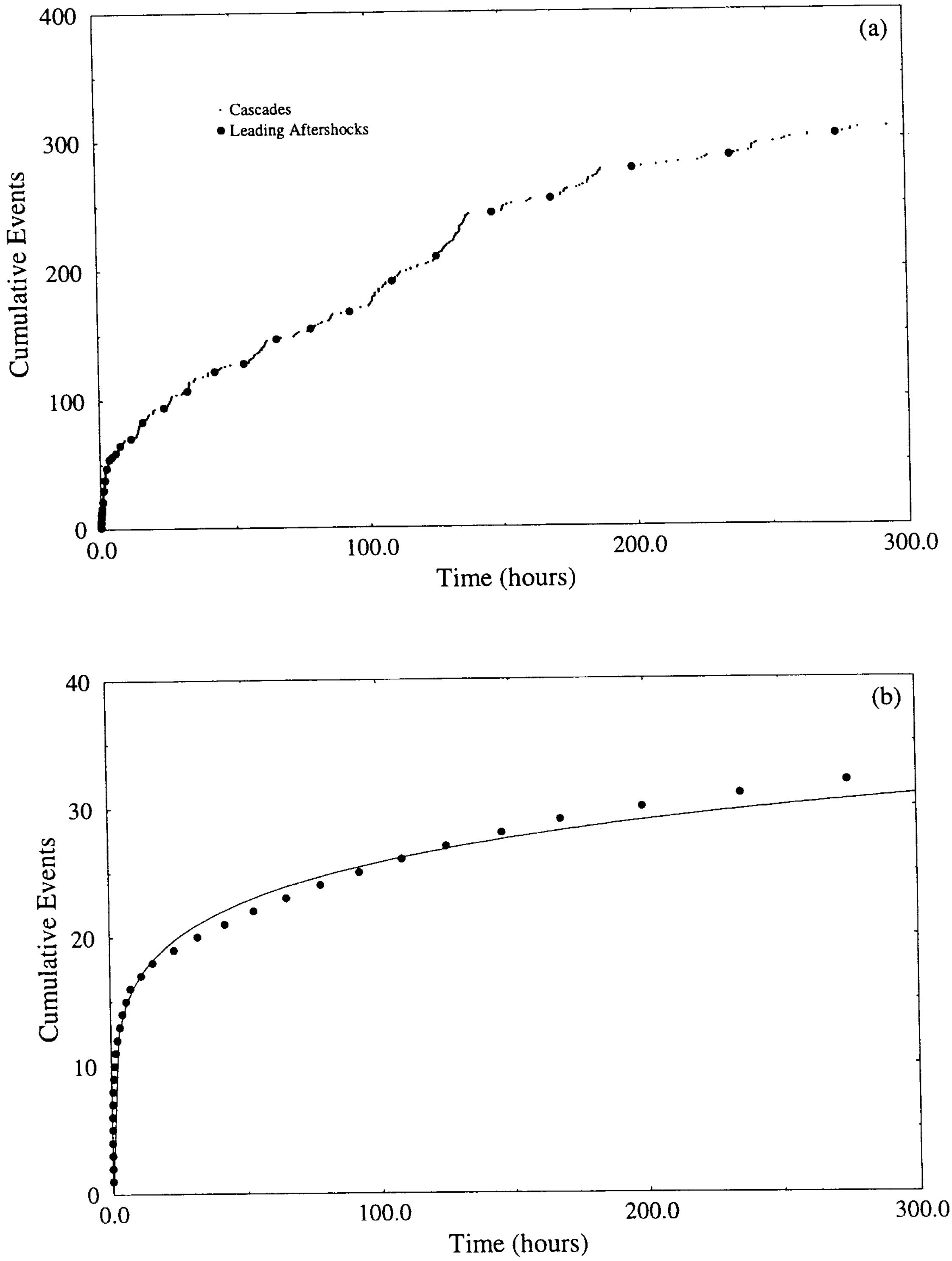

Figure 4. (a) Separation of leading aftershocks and (b) excellent fit of the series of the leading aftershocks to Omori's law.

events that define a cascade can be fitted, in general, to a straight line, thus implying that these events occur at constant velocity.

We have considered a simple approach in order to get some insight into the process of generation of leading aftershocks and cascades, based on the assumption that the rupture of a fault may be thought of as the result of a critical self-organized system (for a recent review of SOC models applied to seismology, see *Main* [1996]).

Chen and Bak [1989] devised a simple toy model to represent the evolution of a dynamical system which evolves to a scale free structure, *i.e.*, to a self-organized critical state. Quoting *Chen and Bak* [1989, p. 299], "we believe that *although the model is not a realistic*

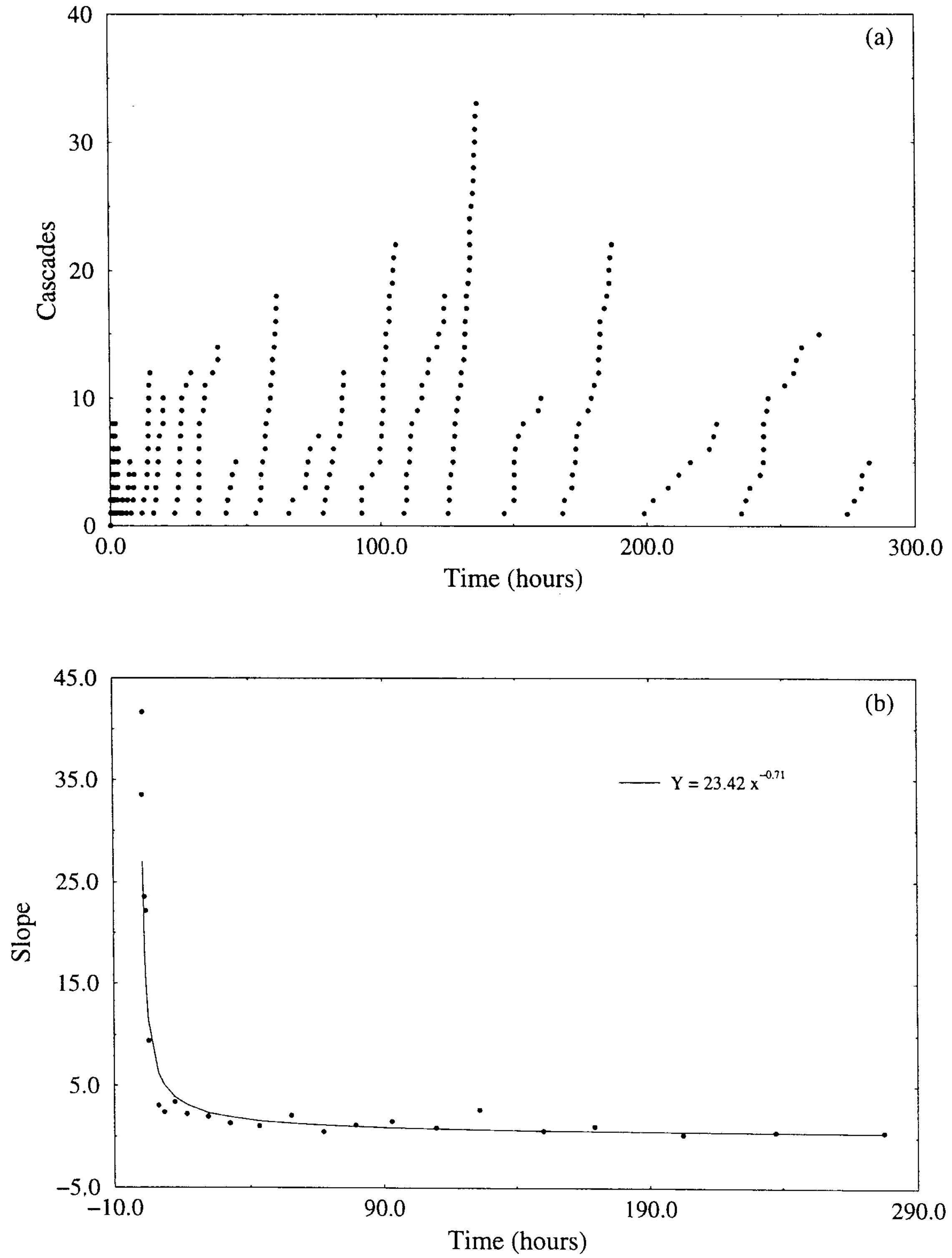

Figure 5. (a) An example of the main cascades retrieved from the original time series. It can clearly be seen that they can be represented by straight lines. (b) Once their corresponding slopes are computed, it can be seen that they follow a potential law.

representation of any particular system, it captures a general scenario for the emergence of scaling behavior, and may provide a guideline for systematically exploring a variety ... of phenomena in nature." We will show that this is indeed the case and that this model can be related to the flow of seismicity through the model of nucleation and origin of seismicity developed by *Cochard and Madariaga* [1994, 1996].

Let us define a cellular automaton consisting of a regular two-dimensional lattice where the cells (sites) might have three possible states: active, passive or empty. The rules for the parallel updating of the system from time t to time $t+1$ are the following: (1) Cells that are active at t burn out and become passive at time $t+1$. (2) Passive cells are annihilated (i.e., become empty) when they have one, and only one, active neighbor. (3) Empty cells become active when they have one, and only one, active neighbor, which must have a passive cell at the opposite position.

Figure 6 displays the rules of the cellular automata, which can also be summarized in the following way: let us define the possible state of the automata as 0, empty cell; 1, passive cell; and 2, active cell. Then the rules are (1) 2 $\longrightarrow$ 1 (independently of the nearest neighbors) (2) 210 $\longrightarrow$ 100 (where step (1) has been used), and (3) 0120 $\longrightarrow$ 0012 (that represents the propagation of the activity in the direction pointed out by the pair 12, having used step (1) in the third position and step (2) in the second).

Following *Chen and Bak* [1989], open boundary conditions have been considered in all cases, so that active cells vanish when passing beyond the edges into the environment. As initial conditions, a random distribution of active and passive cells is used. The system evolves until no active cells remain, and at this point a cell is randomly activated. Figure 7 displays a snapshot of the propagation of the active cells.

The propagation of the active cells through the lattice is similar to a forest fire model, and we will see that this propagation also reproduces quite well the double sequence of leading aftershocks and cascades. Moreover, this minimalist model shares some resemblance with *Cochard and Madariaga*'s [1994, 1996] model. Cochard and Madariaga model the dynamics of the faulting through a rate-dependent friction law. For a highly rate-dependent friction, the nucleation of the fault can become very complex, displaying, among others, the following features of interest for our purposes: (1) premature locking of the fault occurs, so that the slip duration at any point of the fault is independent of the total size of the fault, (2) premature healing is associated with partial stress drop, so that stress heterogeneity may be simply due to the extreme sensitivity of the fault stress to very small changes in the slip distribution, and (3) premature healing is also associated with the generation of self-healing pulses proposed by *Heaton* [1990]. Those properties are quite similar to those of the evolution of the active cells.

With *Cochard and Madariaga*'s [1994, 1996] model in mind, let us define the fault plane as a regular lattice, with each cell representing a small portion of the fault. A cell is an asperity, or a piece of an asperity, for which the stress is higher than a threshold stress, and less than a critical one. When the stress reaches its critical value, the cell becomes active (slip begins with a corresponding stress release, the stress drop), burns out and becomes passive.

Depending on the value of the stress at the neighboring asperities, annihilated cells became active and passive cells may or may not be annihilated, being a possible mechanism for this process a healing phase [*Heaton*, 1990; *Cochard and Madariaga*, 1994]. Hence an annihilated cell can be identified with a cell with a healed slip. Again depending on the neighboring cells, a passive cell may or may not become an annihilated cell at later times.

In short, the equivalence can be stated as follows:

passive cell	$\longleftrightarrow$	broken asperity
active cell	$\longleftrightarrow$	asperity ready to break
annihilated cell	$\longleftrightarrow$	healed cell

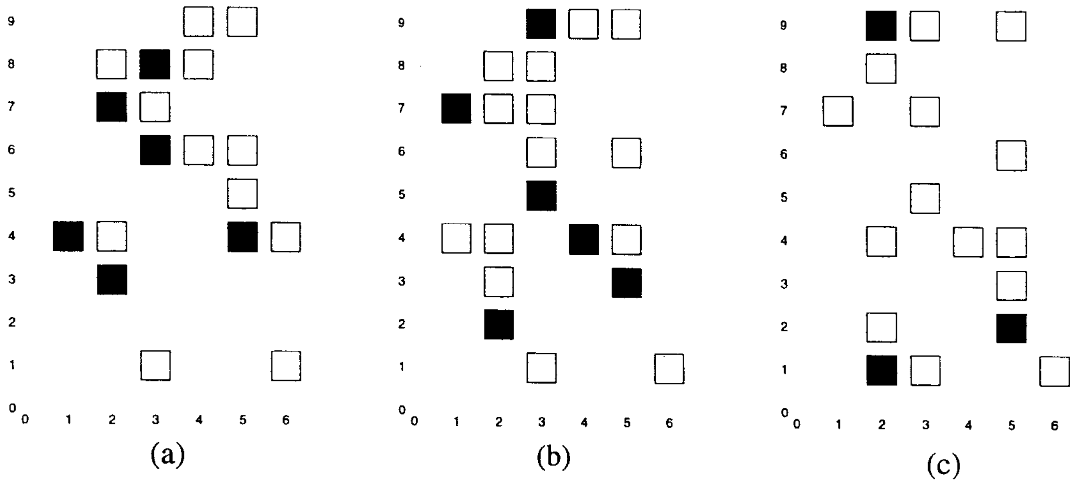

Figure 6. Example of the application in a 2-D model of the rules of the minimalist model. Solid squares refer to an active cell, open squares refer to a passive cell, and the rest to empty cells. The evolution follows from (a) to (c). The empty cell located at (3,5) in Figure 6a, by rule 3 evolves to an active cell in Figure 6b and to a passive cell in Figure 6c due to rule 1. The cell (4,5) is active in Figure 6a, evolves to a passive cell in figure 6b due to rule 1 and remains passive in Figure 6c due to rule 2 (because it has more than one active neighbor). Empty cell (4,3) in Figure 6a remains empty in Figure 6b as well as in Figure 6c due to rule 3 (because it has more than one active neighbor).

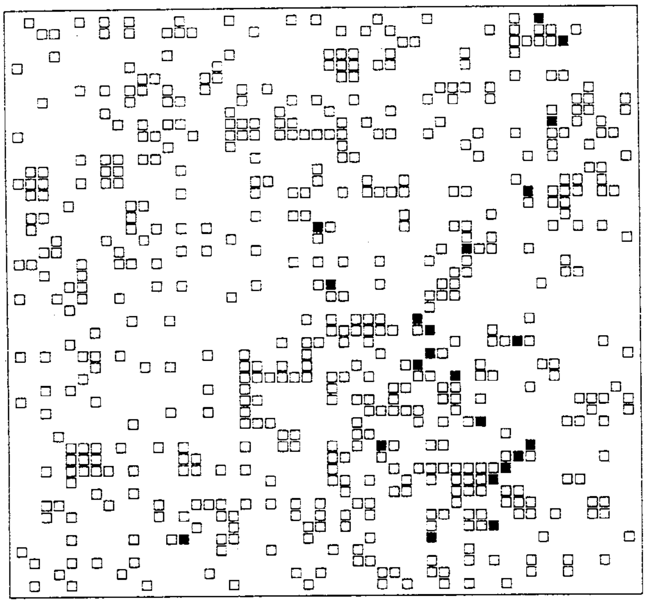

Figure 7. Snapshot of a 50 × 50 grid. Symbols are the same as in Figure 6. The activity can only be propagated to empty cells. Passive cells display spatial clustering.

In the classical studies of SOC models, usually the number of active cells in the system is used as a direct measure of the degree of activity. As dynamics proceeds, the number of active cells decreases, and the system reaches a stable, inactive state. When this happens, a single cell is selected at random and activated. Between two consecutive activations, avalanches may take place instantly, and each avalanche is considered as the sum of the active cells, separated from the next avalanche by a period without activity. The temporal scale is thus defined in terms of activations.

In the present study, we have followed a different approach: we have been looking at the behavior of the system in the sense of recording the number of active cells at each simulation step during the whole period of activity, that is, timescale refers to each step of the simulation, so that the avalanches are extended in time. Figure 8 shows a seismic catalog consisting of 500 activations, once discarded the first 10000, for a 50 × 50 lattice grid. We define a quake as the period between two different activations, and its corresponding intensity is characterized by the total number of activated cells during the whole period of the quake. The units of the abscissa, named seismic series, are the steps of the simulation. Figure 9a displays the temporal evolution of a quake of Figure 8, that of order number 383; the abscissa now refers to simulation (time) steps. This quake starts at about $t_s \simeq 178,000$ and continues for about 3300 timesteps and displays wide fluctuations in the number of active cells.

When dealing with real seismograms, there is always a minimum level of activity required in order to detect the earthquake (for the analyzed series of aftershocks, the threshold detection magnitude was 1.9). We have taken arbitrarily as a detection threshold 25 simultaneously active cells; if the level of activity is less than this figure, the intensity will not be "detected" (i.e., recorded) and will be considered as 0. An event is then defined by the upper curve of active cells delimited by two successive crossing the threshold, as, for example, that defined from approximately 181,500 to 181,900 timesteps. Viewed in this way, the size of the event is

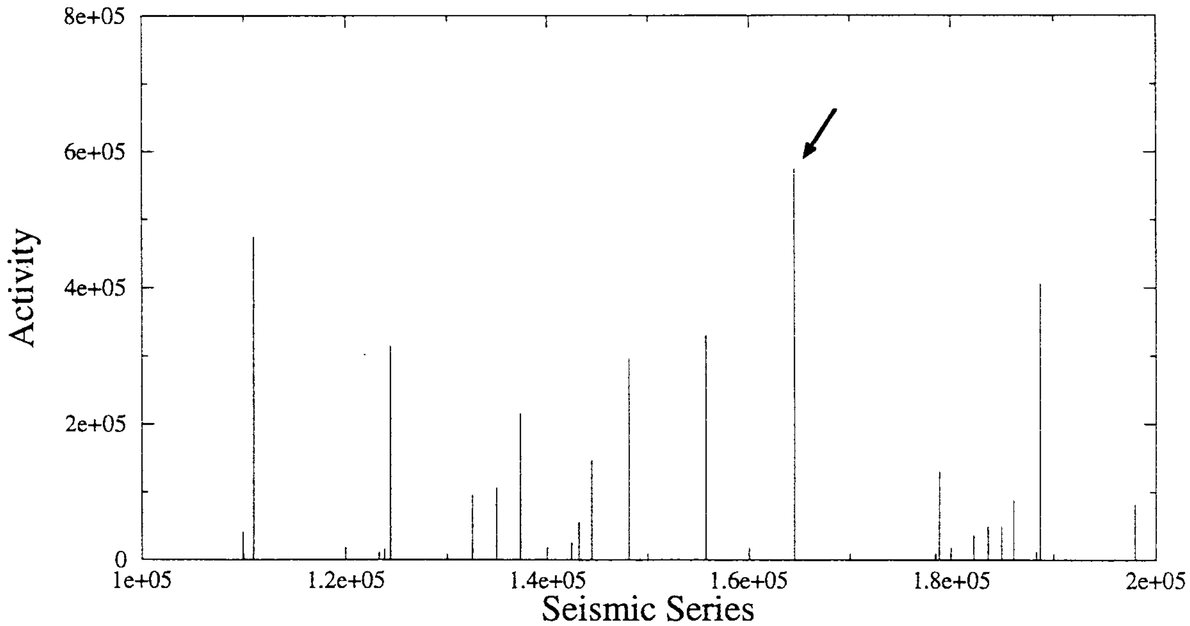

Figure 8. Example of a seismic catalog generated from the minimalist model. Each spike has the meaning of a quake (for more details see the text). The catalog consists of 500 quakes. Read $1e+05$ as 1×10^5.

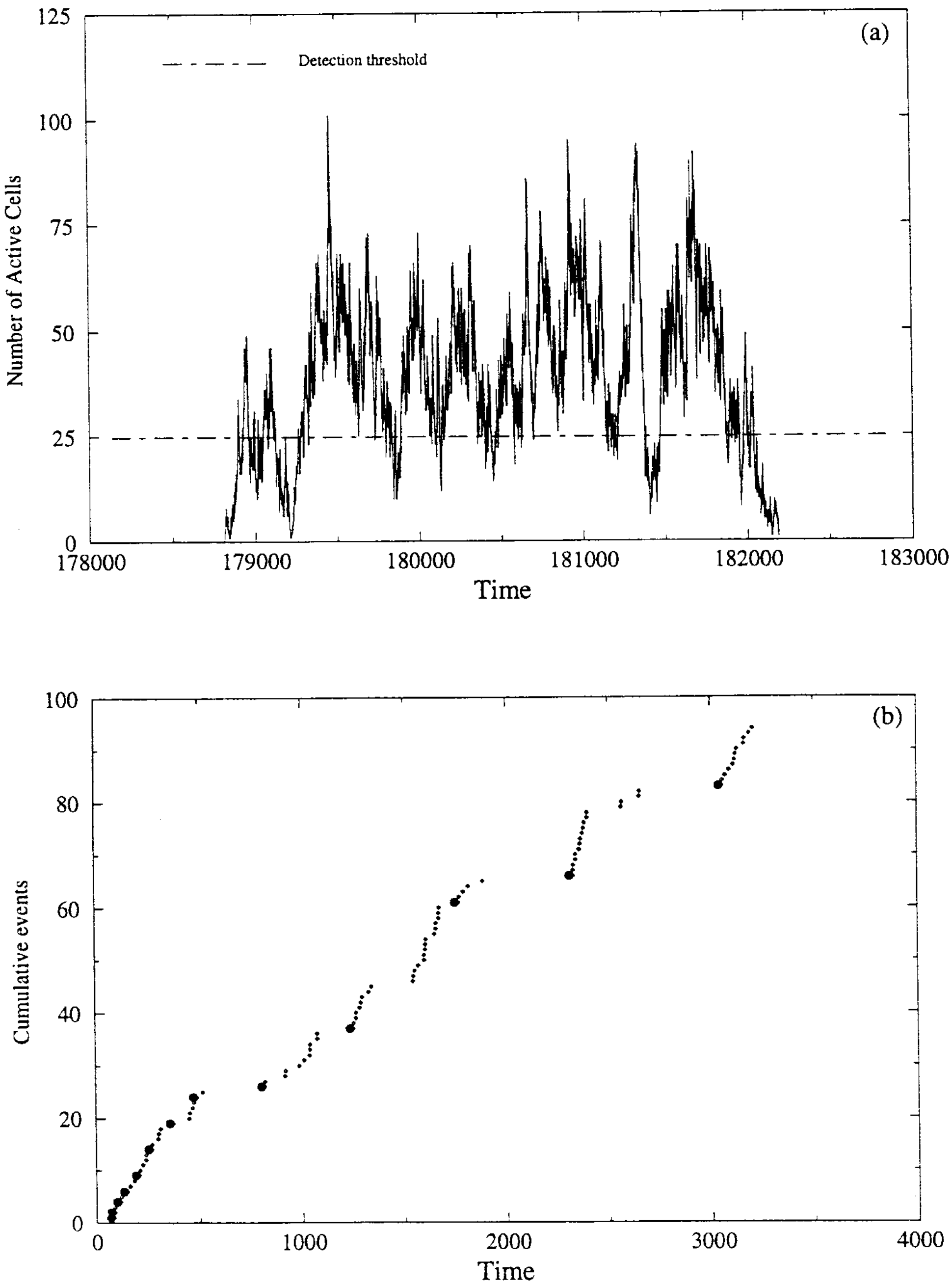

Figure 9. (a) Zoomed view of quake 383 of Figure 8. An event is defined as the upper curve delimited by two zeroes, as, for example, between 181,500 and 181,900 time steps. If a detection threshold is added, the quake is composed of several events, this number depending on the height of the threshold. (b) Cumulative curve of a quake (composed of several events). By comparing with Figure 4a, we can see that this curve is composed of leading aftershocks and cascades.

strongly dependent on the threshold level, but as we will see, the geometrical structure of the cumulative curve (the object of the present study) is preserved.

Let us now analyze a quake (a set of events) in detail, just in the same way as we have analyzed the aftershock series, that is, through the time elapsed between successive events. Figure 9b shows the cumulative number of the successive terms (the number of active cells of an event for each time step) of an event of Figure 9a: Figure 9b displays the same geometric characteristics as Figure 2b, that is, similar changes of concavity. In the present case, the changes of concavity can be explained in terms of the inhomogeneities generated by the dynamical rules of the minimalist model. In fact, passive cells (broken asperities) tend to be spatially clustered, as seen in Figure 7. Then, when a propagating pair susceptible of activation reaches one of those accumulation of asperities, an "aftershock" is triggered, and the increase of local activity translates into an increase in the event rate. A total of 100 simulations have been car-

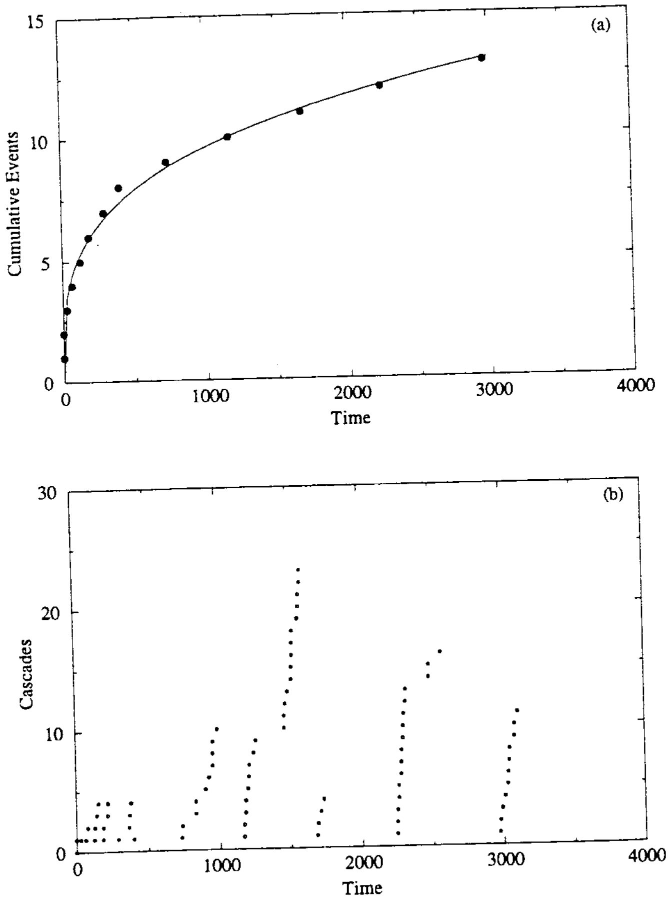

Figure 10. (a) Cumulative curve of the leading aftershocks retrieved from the quake shown in Figure 9b. Note the excellent fit to Omori's law. (b) Cascades induced by each of the previous leading aftershocks. Compare this figure with Figures 4b and 5a.

ried out, with different initial conditions, different grid size (ranging from 25 × 25 to 100 × 100) and different threshold level (ranging from 10 to 50). We have always found the same geometrical characteristics, that is, the changes of concavities, as shown in Figure 9b; the only difference, as expected, is the number of points of the concavities.

The cumulative series, such as shown in Figure 9b, have been decomposed into leading aftershocks and cascades, in the same way as for the observed aftershocks. Figure 10 shows one of such decompositions: leading aftershocks (Figure 10a) and cascades (Figure 10b); they are indistinguishable of the decomposition of the observed series; see Figures 4b and 5a. A fit of the distinct curves of leading aftershocks to the cumulative curve (2) reveals the following values: $k = 0.39$, $c \simeq 0.0$ and $p = 0.72$. We computed the slopes of the cascades for each event, and at the contrary of the series of observed aftershocks, the slope does not fit a potential law, but rather look random. This can be explained because the minimalist model is too simple to take into account the energy propagated as seismic waves, and the loss of energy due to irreversible processes. The cumulative curve (2) has been fitted to the 100 numerical simulations, obtaining for the exponent p the following mean value: $p = 0.7 \pm 0.1$.

A characteristic feature of the numerical simulations is that the events that define the quakes are all of similar amplitude; see Figure 9a. This is also the case for the observed aftershock series, for lapse times greater than 60 hours, see Figure 2b. While this similarity in amplitudes appears to be intrinsic to the minimalist model, we do not know whether it is a general feature of the process of rupture, so that the similarity in both distribution of amplitudes may be fortuitous. The present paper has been concerned only with the geometrical features of the interevent time, so that no efforts have been devoted to the analysis of amplitudes, which will be the subject of a future paper.

6. Discussion and Conclusions

In this paper we have attempted to explain an apparently anomalous aftershock series. Assuming that an aftershock series is a relaxation process, the anomaly consists of sudden increases in the rate of occurrence, not allowed in a strictly relaxing process, without the presence of a large event that would have triggered secondary series. The detailed view of this increase was possible because of the low detection threshold of the CAD broadband seismic station, allowing the detection of events of magnitude 1.9 for an epicentral area 80 km apart. After a classification of the events into leading aftershocks and cascades, it has been found that the leading aftershocks obey a power law relaxation, that is, Omori's law, whereas cascades occur at a nearly constant rate. This process could be interpreted in terms of a nucleation characterized by two different friction laws: rate dependent and constant rate, the former accounting for the initiation of the nucleation of an asperity and the latter for the rupture of the asperity itself.

Evidence in favor of this interpretation is the correct prediction of the return time of the main shock from the observed time series of leading aftershocks. On the other hand, this result would imply that *Dietrich*'s [1994] model is able to take into account the rate of rupture of asperities but not the rupture mechanism of the asperity itself. Because of the errors in epicentral location, the evolution of the rupture in the fault plane cannot be measured. Thus we have to rely on theoretical models, as, for example, that of *Cochard and Madariaga* [1994]. In that model the time evolution of the rupture is closely related to the evolution of the (inhomogeneous) accumulation of stress in the source region.

We propose that some features of the dynamical behavior of Cochard and Madariaga's model, such as the evolution of the stress concentration and evolution of rupture, can be retrieved from the SOC models so frequently used for the simulation of seismic catalogs. In SOC, an open system evolves to a stationary state, called critical state. A possible explanation for reaching this state could be the accumulation of stress from an external source. If at some point of the system the accumulated stress is higher than a given threshold, the stress is released in terms of avalanches of all sizes. When trying to explain the observed aftershock series, we are interested not in the global behavior of the system but in the internal structure of a single event, which we have termed a quake. The occurrence of the quake can be thought of as a certain number of superpositions of relaxation processes, each one corresponding to an avalanche. Once the system is activated, it evolves until the excess of stress is completely released.

We have studied the temporal evolution of an aftershock series through a minimalist model that qualitatively resembles *Cochard and Madariaga*'s [1994, 1996] model of nucleation if instead of passive, active and empty cells, we translate to unbroken asperities, asperities ready to break and healed cells. This simple SOC model correctly predicts the behavior of the interevent occurrence time of the observations, but more evidence is needed to claim that we are indeed in the presence of a SOC phenomenon. Further evidence in favor of SOC is provided by *Cochard and Madariaga* [1996] in their comment about rupture propagation governed by a nonlinear rate-dependent friction law. They state that the rupture "adjusts itself" in order to satisfy a scaling law, which suggests the presence of an internal feedback in the system, very often responsible for the appearance of self-organized critical states [*Sornette*, 1992].

Another aspect that deserves some comment is that of the amplitudes. Up to now we have discussed the series of observed aftershocks as a point series in time axis, but nothing has been said about their amplitude. We have found that the leading aftershocks do not have larger magnitudes than the rest of events. This implies that small events can trigger cascades of similar amplitudes. This observation is of interest because it implies that small events may precede large events, as in the case of foreshocks. Hence in this kind of relaxation process the size of the events decreases on the average but might be strongly affected from fluctuations. However, although the minimalist model reproduces many of the observed features in field measures, we have to be well aware that the minimalist model is a toy model, that is, not a detailed representation of the physical process, but anyway, a model that captures a general scenario for the emergence of scaling behavior [*Bak and Tang*, 1989].

The present work can be summarized as follows:

1. We have analyzed an anomalous behavior in the time occurrence of aftershocks. This behavior had not been previously detected due to the fact that it is necessary to combine a very low level of detectability of the seismic stations, along with the proximity of the epicentral area.

2. We have interpreted the relaxation process implicit in the observations in terms of a continuous model, and we have recovered a realistic return time for the main shock.

3. A similar relaxation process has been found in a simple SOC system.

4. The SOC model we have used has a physical basis in the nucleation model developed by *Cochard and Madariaga* [1994].

As a consequence, more efforts will be devoted to study the dynamical characteristics of the model (i.e., the temporal evolution of the amplitudes), the influence of some dissipation on the model, and the possible spatial clustering properties (S. C. Manrubia, R. V. Solé, M. Urquizú, and A. M. Correig, Fractality and Aftershocks in a SOC Model for Earthquakes, manuscript in preparation, 1997). It is our feeling that the minimalist model, if able to explain the dynamic characteristics, may be a useful tool in studies of probabilistic prediction.

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