

**Zanette and Manrubia Reply:** As Marsili *et al.* [1] point out, the average local population in our model is preserved,  $\langle n_i(t) \rangle = \text{const}$ . This ensures that in an infinite system the total population  $N = \sum_i n_i(t)$  is strictly preserved, and fluctuations do not play any role. In our model, fluctuations govern the spatial distribution of population, as discussed in the Letter, but are irrelevant—in the infinite population limit—to the temporal evolution of  $N$ . The irrelevance of fluctuations in the infinite-size limit of Zeldovich models with diffusion (the class of systems to which our model belongs) has already been discussed, for instance, in Ref. [12] of our Letter. Notably enough, in a paper by one of the authors of the Comment [Y. C. Zhang *et al.*, *J. Stat. Phys.* **58**, 849 (1990)], the decreasing effect of fluctuations as the system size is increased has been explicitly proven for a birth-death stochastic model very similar to ours. There, it is shown that the typical extinction time of a population with identical birth and death rates is of the order of  $\sqrt{N}$ , i.e., diverges for infinite populations. In the limit, the population remains constant in time. In the Comment, the authors fail to indicate how the extinction time  $\langle \ln F \rangle^{-1}$  depends on the population size, which is extremely misleading to the interpretation of their argument.

It is clear that the analytical results of our Letter hold for infinite populations, since fluctuations are not taken into account in our interpretation of probabilities as frequencies in infinite series of realizations. On the other hand, our numerical simulations—which involve of course finite systems—had to be corrected to take into account finite-size effects, which are a standard problem in simulations of stochastic processes with birth-death events (see, for instance, Ref. [22] of our Letter). The correction method, which is similar to the “source terms” proposed at the end of the Comment, is explained in detail in Ref. [23] of our Letter.

It is worthwhile to stress that even if this correction is not introduced, simulations in large systems show clearly a transient in which a distribution with the expected power law  $n^{-2}$  builds up, before finite-size effects play their role. To illustrate this, we have performed additional numerical simulations *without* the above mentioned correction, on a  $(100 \times 100)$ -site lattice. Figure 1 shows the temporal average of the population distribution, performed during times before extinction (with an additional average over some 20 realizations). We have also verified that the extinction time grows as the size increases, as mentioned above. Along with the analytical results of our Letter, these additional simulations *without* corrections strongly suggest that the power-law decay of the population frequency,  $n^{-2}$ , characterizes the asymptotic distribution in the model.

Another point that is intimately related to the interpretation of our results has to do with the combined effect of diffusion and reaction events, that—as we pointed out

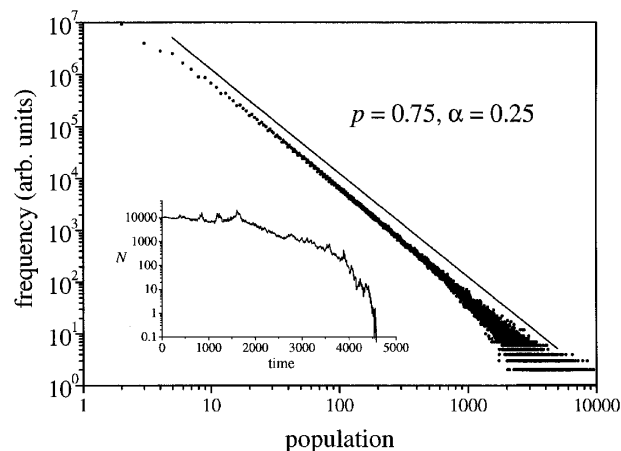


FIG. 1. Temporal average (before extinction) of the population frequency in simulations *without* finite-size corrections, on a  $(100 \times 100)$ -site lattice. The straight line has slope  $-2$ . Inset: Evolution of the total population. Both plots stand for averages over some 20 realizations.

in our Letter—determine the stationarity of the population distribution. The only case worked out in the Comment is the diffusionless one,  $\alpha = 0$ . The authors then assert (without proof) that diffusion ( $\alpha > 0$ ) alone cannot reverse the decay that, they claim, affects the total population. This is not true. First of all, the case  $\alpha = 0$  can be exactly solved in the limit of an infinite population. This is shown, for instance, in Ref. [21] of our Letter. In such a case, as is well known for any multiplicative stochastic process, the resulting population distribution—which does preserve the total population—is log normal and nonstationary. As also shown in Ref. [21] of our Letter, however, adding a transport process to the reactionlike events can drastically modify the exponent of this decay and make the distribution stationary. This is also implicit in Refs. [3] and [4] of the Comment, where the source terms added to prevent extinction in finite systems can be thought of as representing a kind of transport mechanism that brings population to the system. The diffusionless case analyzed in the Comment is therefore completely marginal, and there should be no surprise in the fact that diffusion mechanisms can affect the exponent of the population distribution.

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[1] Matteo Marsili, Sergei Maslov, and Yi-Cheng Zhang, preceding Comment, *Phys. Rev. Lett.* **80**, 4830 (1998).